By (iii) there is a c-isomorphism $dX/dY \to G$. Meanwhile d induces an isomorphism $X/Y \to dX/dY$, and so, modulo the first and second isomorphism theorems, an isomorphism $F \to G$, as desired.

The converse is immediate.

Property (v). Suppose we are given distinct regions N, N' to which any $\mathfrak{A}$ attaches isomorphic groups. We shall show that they must be δ-paired. In other words if λF, λG are neither equal nor δ-paired, and $\mathfrak{A}$ is sufficiently general, then $F \ncong G$.

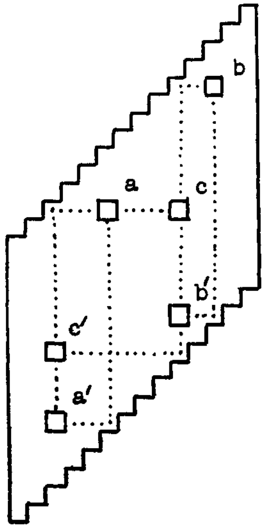

FIG. 7

Since N, N' are distinct, either $N - N'$ or $N' - N \neq \emptyset$. Suppose $a \subset N - N'$. Then $a \not\subset \Delta^0$, by Lemma 12 (i). Therefore a has a δ-pair, a' say; by Lemma 12 (ii)

(1) if $a \subset N - N'$ then $a' \subset N' - N$.

Also

(2) $N \cap \Delta^0 = \emptyset$,

for suppose $b \subset N \cap \Delta^0$. Then either $b \not\subset N'$ contradicting Lemma 12 (i), or $b \subset N'$ contradicting Lemma 12 (iii).

(3) If $a \subset N \cap \Delta^+$ and $b' \subset N \cap \Delta^-$ then $a \not> b'$,

otherwise there exists $c \subset \Delta^0$, $a > c > b'$, which is contained in N by Lemma 3, contradicting (2). We now deduce the stronger statement

(4) N cannot meet both Δ^+ and Δ^-.

For let $a \subset N - N'$, and its δ-pair $a' \subset N' - N$ by (1). We may suppose without loss of generality that $a \subset \Delta^+$, and so $a' = \delta a$. Assume the converse of (4) that $b' \subset N \cap \Delta^-$. Let $\delta b = b'$. From (3) $a \not> b'$, and by Lemma 11, $a < b$. Therefore $a' < a < b$, $a' \subset N'$, and so $b \not\subset N'$ by (3). Hence $b \subset N - N'$, $b' \subset N' - N$ by (1), implying $b' \subset N'$. Let c be the least upper bound of a and b', and let

and represent the transgressive elements of dimension n in the base, and dimension $n - 1$ in the fibre, respectively. The fact that they occur in the middle of $H_n(B)$ and $H_{n-1}(F)$ indicates, by Lemma 4 that we cannot in general extend the "isomorphism" ${}^n\delta: {}^n[n, 1] \leftrightarrow {}^{n-1}[0, -1]$ either way into a homomorphism between $H_n(B)$ and $H_{n-1}(F)$. However ${}^n[n, 1]$ does occur at the top of λP_n, where P_n is the subgroup of $H_n(B)$ corresponding to the two lowest squares; and ${}^{n-1}[0, -1]$ occurs at the bottom of λQ_{n-1}, where Q_{n-1} is the quotient group of $H_{n-1}(F)$ corresponding to the two highest squares. Now we can apply Theorem 5 and obtain the transgression

$$P_n \to Q_{n-1}$$

induced by d. Alternatively, if $H_n(E) = H_{n-1}(E) = 0$, then ${}^n[n, 1]$ occurs at the bottom and ${}^{n-1}[0, -1]$ at the top, enabling us to define the suspension

$$H_n(B) \leftarrow H_{n-1}(F)$$

induced by d^{-1}.

We denote by ${}^n\Gamma$ the heavily outlined region of ${}^n\Delta$ in Figure 8. Its full significance will appear in the next section. Meanwhile we illustrate its use by considering the situation when the fibre is *totally non-homologous to zero*. This is described algebraically by $E^2 = E^\infty$, and, on the diagram, by attaching zero groups to all the squares contained in ${}^n\Gamma \cap ({}^n\Delta^+ \cup {}^n\Delta^-)$. As a result we see that i_* becomes a monomorphism, and h_* an endomorphism.

We conclude this descriptive section by mentioning that similar diagrams can be drawn for cohomology, but that they do not display the multiplicative structure.

5. The invariant subcategory

Suppose the three spaces of a fibre space are polyhedra. The terms E^r, $r \geqq 2$, of the associated spectral sequence are "invariant" in the sense that they are independent of the method of calculation, whether it be by singular cubical homology (Serre [8]), by singular simplicial homology, by the use of sheaves (Leray [5]), or by dihomology, [9]. We seek for all such invariant groups in $\mathfrak{A}^\#$. In order to preserve simplicity, we return to the discussion of an (ungraded) filtered differential group of length m, as in Section 1, and manufacture a suitable definition of invariance. The generalization to the graded case presents no difficulties and is left to the reader.

DEFINITION, A *homomorphism* $f: \mathfrak{A} \to \mathfrak{A}'$ between two filtered differential groups of length m is a homomorphism $f: A \to A'$ which preserves the structure, that is $fA_p \subset A'_p$, each p, and $fd = d'f$.

If ψ is a formula defining a group of $\mathfrak{A}^\#$ (in terms of the A_p and d), then f induces a homomorphism $\psi(f): \psi(\mathfrak{A}) \to \psi(\mathfrak{A}')$. The term E_p^r of the spectral sequence is an example of such a formula. Define ψ to be *invariant*[5] if $\psi(f)$ is an

[5] If we were interested in a spectral sequence from only the r^{th} term onwards, we could equally well replace E^2 by E^r in the definition of invariance, and throughout Section 5.