THE GENERALISED POINCARÉ CONJECTURE

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THEOREM. *If a combinatorial n-manifold has the homotopy-type of an n-sphere then it is homeomorphic to an n-sphere, provided $n \geqq 5$.*

The above theorem was proved for $n \geqq 7$ by Stallings [2]. His proof can be adapted to cover the cases $n=5$, 6 by means of the following lemma (the proof of which is given in [3]).

LEMMA. *Suppose M^n is a q-connected combinatorial n-manifold, where $q \leqq n-3$. Suppose A^q is a q-subcomplex, and B a collapsible subcomplex, both contained in the interior of M^n. Then there exists a collapsible subcomplex C in the interior of a suitable subdivision σM^n of M^n, such that $C \supset \sigma(A^q+B)$ and* $\dim(C-\sigma B) \leqq q+1$.

The lemma is useful in a variety of contexts. For the application that we need here, choose A^q to be the q-skeleton of M^n and B to be a point; then a regular neighbourhood of C is an n-ball containing