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On Contractible Open Manifolds

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Mathematical Proceedings of the Cambridge Philosophical Society / Volume 58 / Issue 02 / April 1962, pp 221 - 224
DOI: 10.1017/S0305004100036434, Published online: 24 October 2008

Link to this article: http://journals.cambridge.org/abstract_S0305004100036434

How to cite this article:
D. R. McMillan and E. C. Zeeman (1962). On Contractible Open Manifolds. Mathematical Proceedings of the Cambridge Philosophical Society, 58, pp 221-224 doi:10.1017/S0305004100036434

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[redacted]

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Received 8 September 1961

By an open manifold we mean a non-compact space, that is triangulable by a countable complex which is a combinatorial manifold without boundary (see next [redacted]

[redacted]

[redacted]

THEOREM. *If M^n is a contractible open manifold, then $M^n \times E^2$ is piecewise linearly** [redacted]

[redacted]

[redacted]

For example, think of a simple closed curve (of codimension 2) in a solid torus, that is inessential but links itself around the torus, and is therefore non-trivial. Similarly one can construct an n-sphere S^n (of codimension $n+1$) in $S^1 \times B^{2n}$ that is inessential and non-trivial by linking two S^n's locally, and then connecting them by

a pipe round the S^1. For a more detailed discussion, see Zeeman (12).

In a contractible open manifold, of course, every subspace is inessential. If the manifold is not a Euclidean space then it contains a finite combinatorial subspace of codimension at least one which is inessential but non-trivial (see Lemma 4 and § 3). In most examples there is such a subspace which is geometrically significant. For

LEMMA 2. *Let M be a finite combinatorial manifold. Given a combinatorial subspace ${}^{r+1}X \simeq 0$ in M°, then there exist combinatorial subspaces ${}^{r}Y$, ${}^{2r}Z$ in M° such that $X \subset Y \searrow Z$.*

[redacted]

is simple. Y is a cone on X mapped into general position, with singularities of codimension $2r$. We can collapse Y onto the $(2r-1)$-codimensional subcone that contains these singularities. The last step down to codimension $2r$ is achieved by piping the middles of the singularities over the edge of the cone.

LEMMA 3. *Suppose $\{M_i\}$, $i = 1, 2, \ldots$, is a sequence of finite combinatorial n-manifolds, such that each M_i is a combinatorial subspace of M°_{i+1}, and $M_i \simeq 0$ in M°_{i+1}. If ${}^{r+1}X \subset M_i$,*

[redacted]

Now T_i is of codimension 3 in $M_i \times D_i$, which is of dimension $n+2$. Therefore by Lemma 3, T_i is trivial in $(M_{i+n} \times D_{i+n})^\circ$. But $T_i \nearrow M_i \nearrow M_i \times D_i$. Therefore by Lemma 1, there exists an $(n+2)$-ball B_i, such that

$$M_i \times D_i \subset B_i \subset (M_{i+n} \times D_{i+n})^\circ.$$

Taking the union over all i,

$$M \times E^2 \subset \bigcup B_i \subset M \times E^2.$$

Hence $M \times E^2 = \bigcup\limits^{\infty} B_i = \bigcup\limits^{\infty} B_{jn}$, which is the union of a sequence of balls each in the interior of its successor, and which therefore $= E^{n+2}$ by Lemma 4.

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