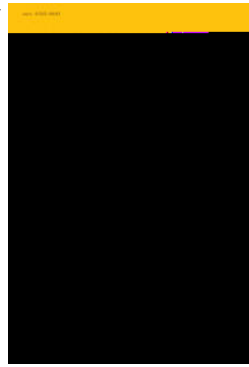


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ON CONTRACTIBLE OPEN MANIFOLDS

BY D. R. McMILLAN AND E. C. ZEEMAN

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By an open manifold we mean a non-compact space, that is triangulable by a countable complex which is a combinatorial manifold without boundary (see next section). The obvious example is Euclidean n -space, which we denote by E^n . We prove:

THEOREM. *If M^n is a contractible open manifold, then $M^n \times E^2$ is piecewise linearly* homeomorphic to E^{n+2} .*

For example, think of a simple closed curve (of codimension 2) in a solid torus, that is inessential but links itself around the torus, and is therefore non-trivial. Similarly one can construct an n -sphere S^n (of codimension $n + 1$) in $S^1 \times B^{2n}$ that is

a pipe round the S^1 . For a more detailed discussion, see Zeeman (12).

In a contractible open manifold, of course, every subspace is inessential. If the manifold is not a Euclidean space then it contains a finite combinatorial subspace of codimension at least one which is inessential but non-trivial (see Lemma 4 and § 3). In most examples there is such a subspace which is geometrically significant. For

LEMMA 2. Let M be a finite combinatorial manifold. Given a combinatorial subspace ${}^{r+1}X \simeq 0$ in M° , then there exist combinatorial subspaces ${}^rY, {}^{2r}Z$ in M° such that $X \subset Y \setminus Z$.

is simple. Y is a cone on X mapped into general position, with singularities of codimension $2r$. We can collapse Y onto the $(2r - 1)$ -codimensional subcone that contains these singularities. The last step down to codimension $2r$ is achieved by piping the middles of the singularities over the edge of the cone.

LEMMA 3. Suppose $\{M_i\}, i = 1, 2, \dots$, is a sequence of finite combinatorial n -manifolds, such that each M_i is a combinatorial subspace of M_{i+1}° , and $M_i \simeq 0$ in M_{i+1}° . If ${}^{r+1}X \subset M_i$,

Now T_i is of codimension 3 in $M_i \times D_i$, which is of dimension $n+2$. Therefore by Lemma 3, T_i is trivial in $(M_{i+n} \times D_{i+n})^\circ$. But $T_i \nearrow M_i \nearrow M_i \times D_i$. Therefore by Lemma 1, there exists an $(n+2)$ -ball B_i , such that

$$M_i \times D_i \subset B_i \subset (M_{i+n} \times D_{i+n})^\circ.$$

Taking the union over all i ,

$$M \times E^2 \subset \bigcup B_i \subset M \times E^2.$$

Hence $M \times E^2 = \bigcup_i B_i = \bigcup_n B_{j_n}$, which is the union of a sequence of balls each in the

interior of its successor, and which therefore $= E^{n+2}$ by Lemma 4.

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