

Now a regular neighbourhood D of C is characterised by three properties: D is a manifold, a neighbourhood of C , and $D \searrow C$ (D collapses to C). It transpires that only the third property matters

first two properties is to make $D - C$ of minimal dimension. In fact we have an equivalent definition: X can be engulfed from C if there exists a PL subspace D such that

$$X \subset D \searrow C,$$

$$\dim(D - C) \leq x + 1.$$

Intuitively one thinks of $D - C$ as a feeler pushed out from C so as to engulf X . The advantage of having the feeler of dimension only one more than that of X is apparent when engulfing singularities of maps is considered. For example, the feeler itself may introduce new singularities, but these will be of lower dimension than what we started with, and so amenable to attack by induction. The need for progressive engulfing (by induction) raises problems in Stallings'

$$\begin{aligned}
 & X \subset D \setminus C, \\
 & \dim(D - C) \leq x + 1, \\
 & \text{---}
 \end{aligned}$$